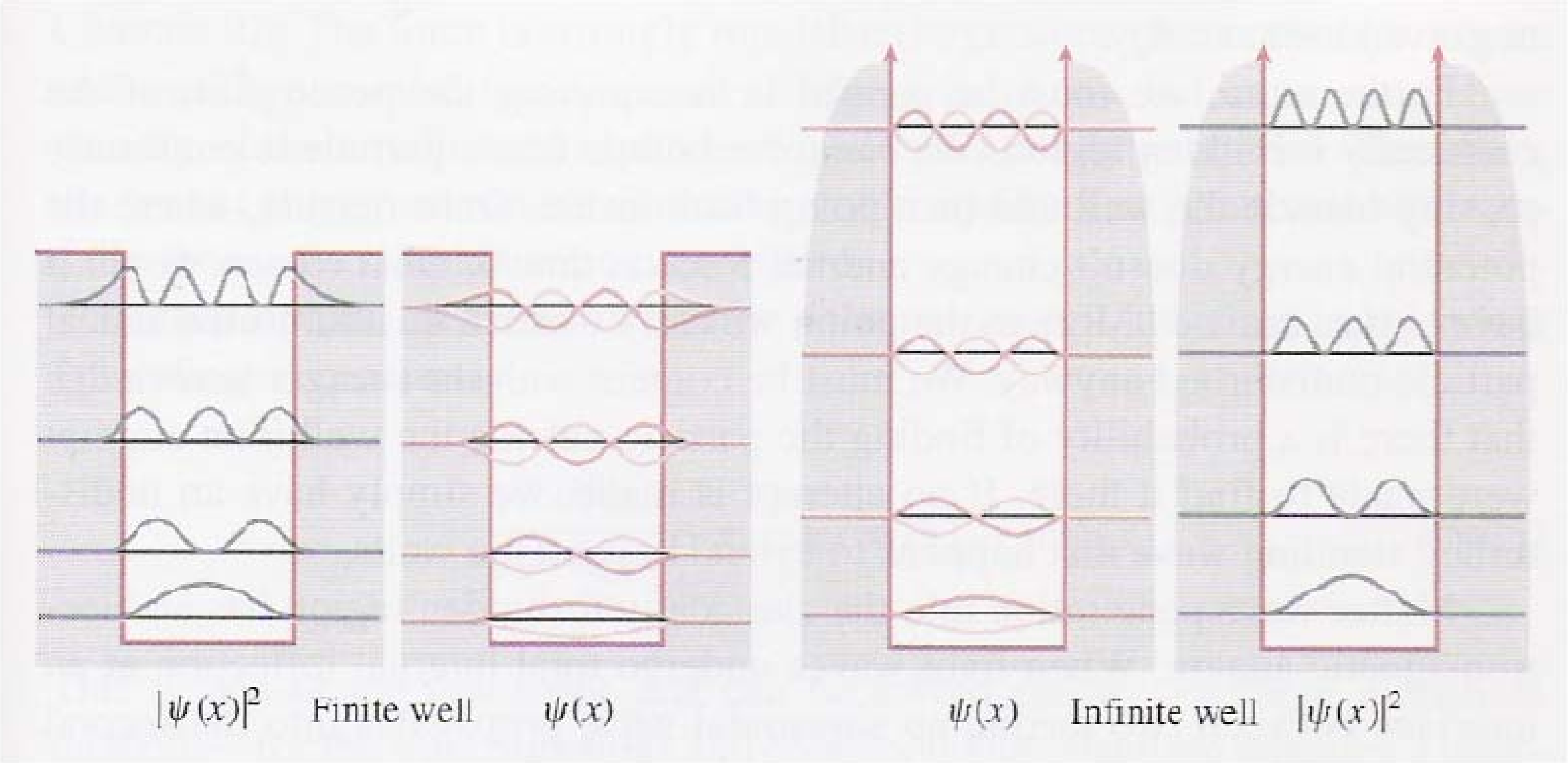
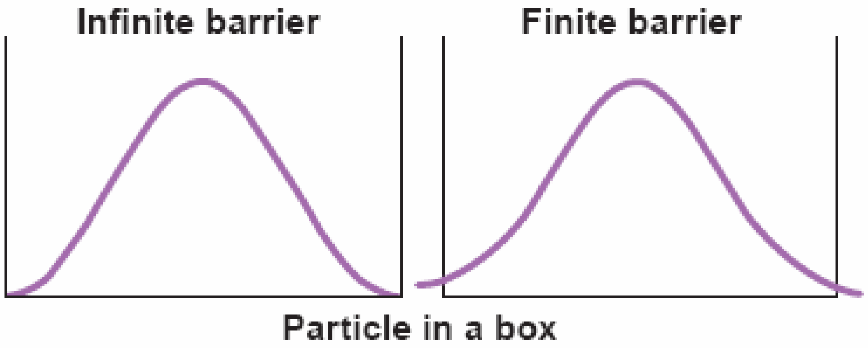
Finite Wells and Quantum Tunneling

Make sure that you are able to set up the Schrödinger equations for the three regions that we discussed for a finite potential well!

Finite vs Infinite Well





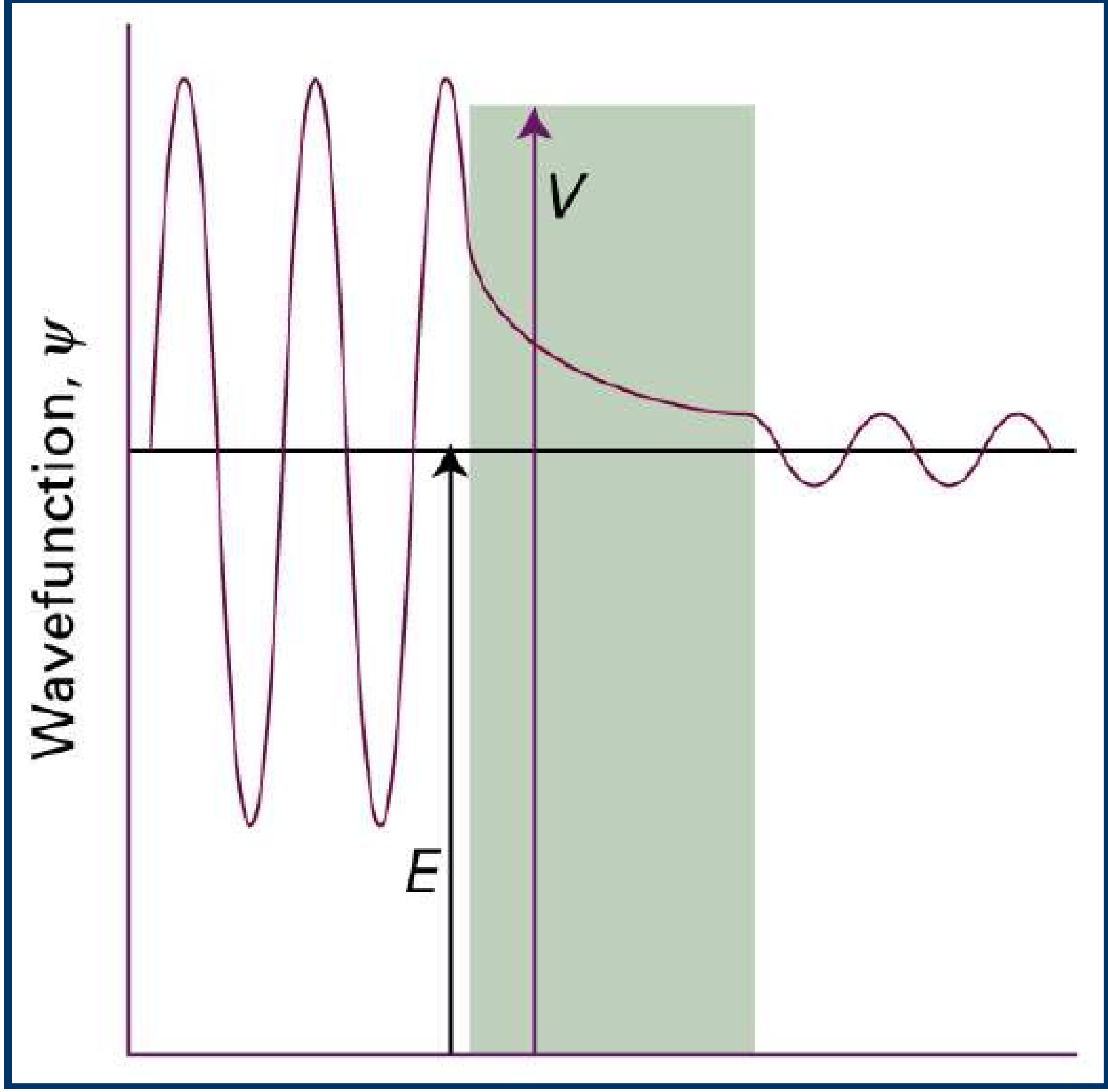


Fig. 8.8 A particle incident on a barrier from the left has an oscillating wave function, but inside the barrier there are no oscillations (for $E < V$). If the barrier is not too thick, the wavefunction is nonzero at its opposite face, and so oscillates begin again there. (Only the real component of the wavefunction is shown.)

The slopes of wavefunctions (their first derivatives) must also be continuous at the edges of the barrier (at $x = 0$ and $x = L$)

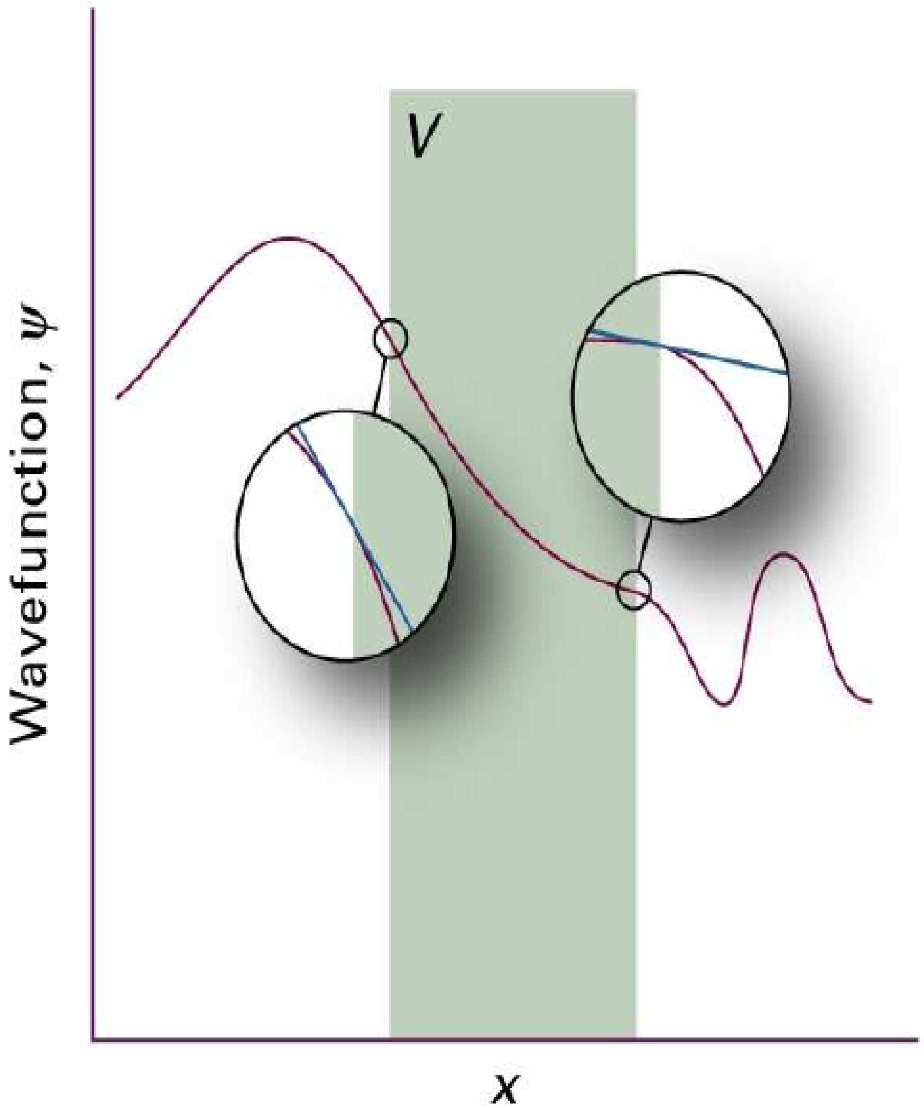


Fig. 8.10 The wavefunction and its slope must be continuous at the edges of the barrier. The conditions for continuity enable us to connect the wavefunctions in the three zones and hence to obtain relations between the coefficients that appear in the solutions of the Schrödinger equation.

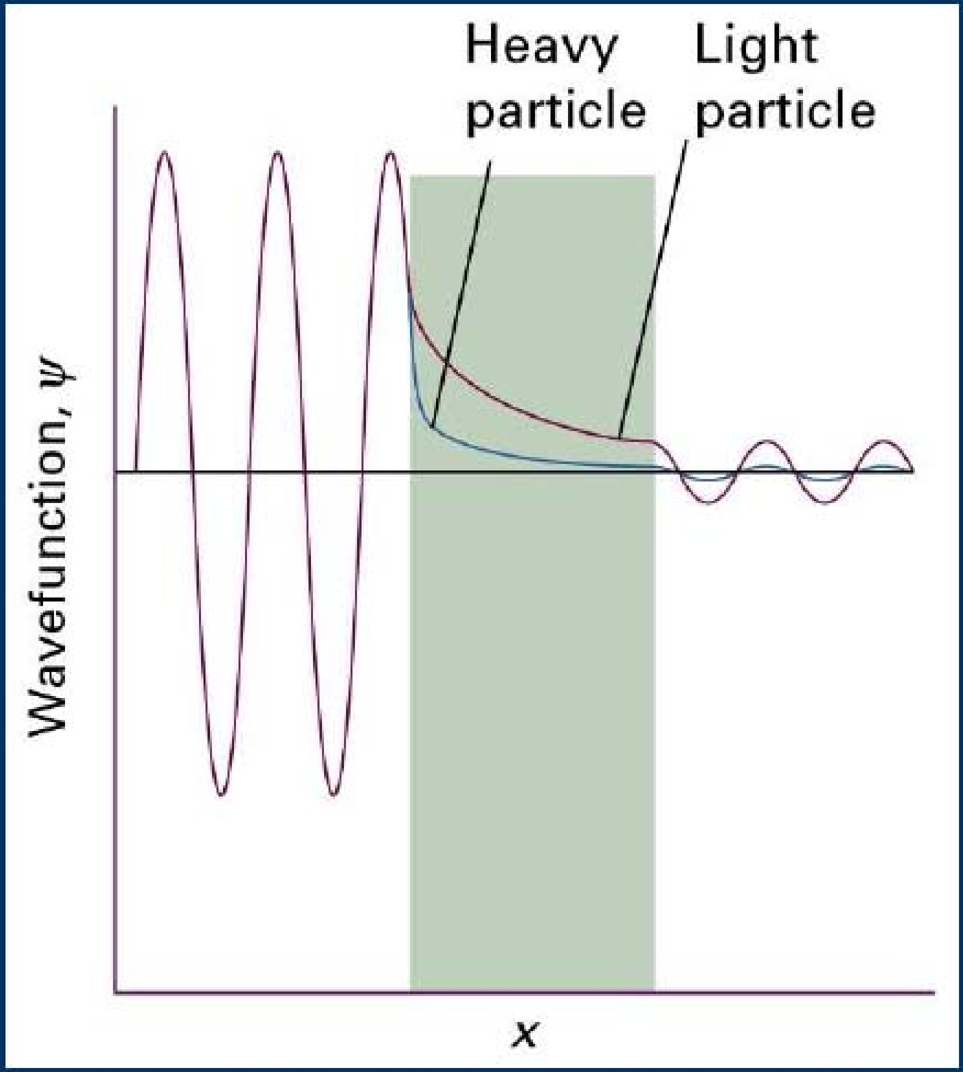
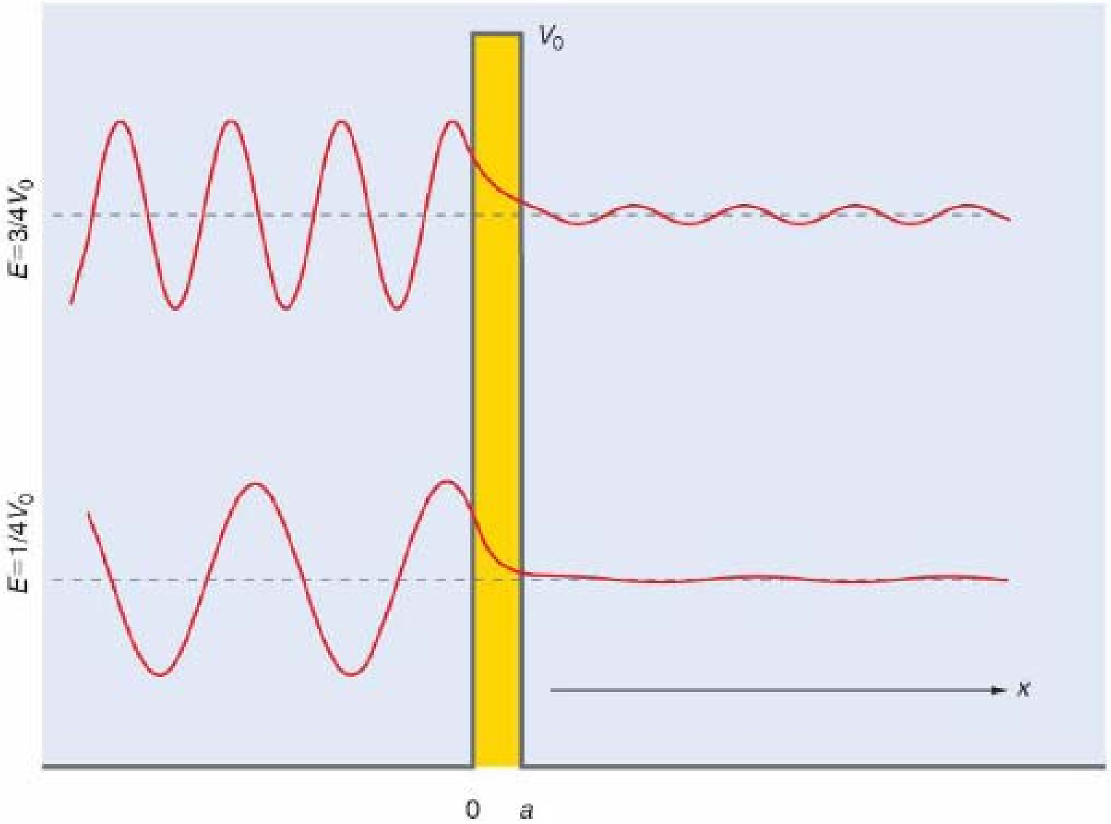
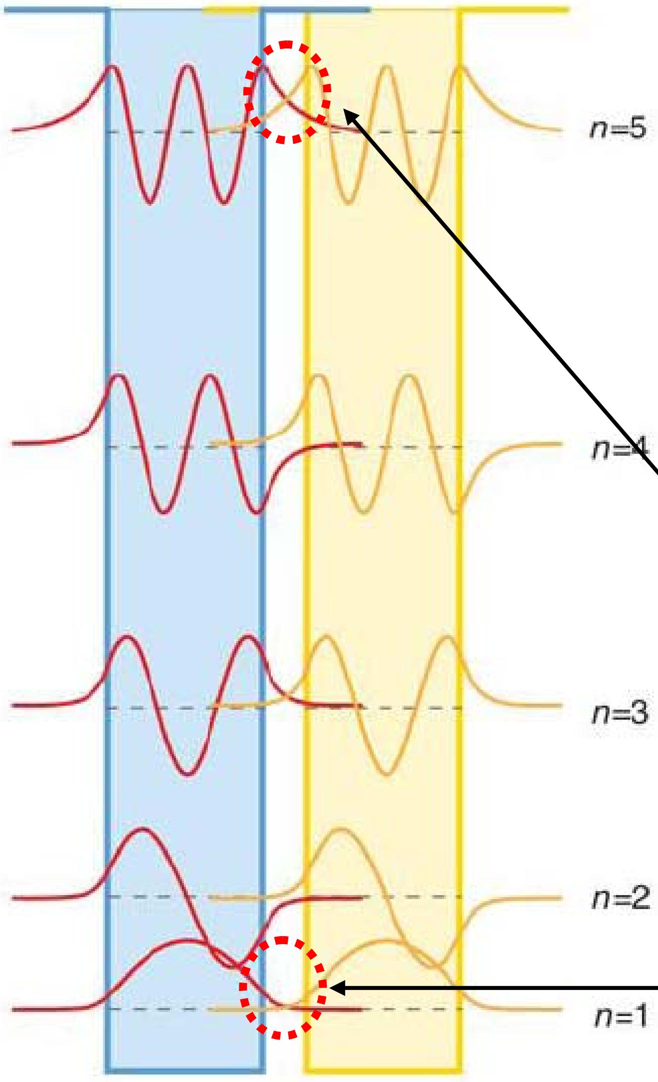


Fig. 8.12 The wavefunction of a heavy particle decays more rapidly inside a barrier than that of a light particle. Consequently, a light particle has a greater probability of tunnelling through the barrier.



Wave function decays into the barrier : $e^{-\kappa x}$ term
$1/\kappa$: decay length
Probability of high-energy particle > probability of low-energy particle

Differences in Overlap between Core and Valence Electrons



Overlap of wave functions n=1 to n=5

Except to n=5, there are almost not overlapping each other. However, when n=5, two wave functions are significant overlapping.
→ make a 'bonding'

Significant Overlap

Small Overlap

Scanning Tunneling Microscopy

Inventors of STM

The Nobel Prize in Physics 1986

Nobel Laureates Heinrich Rohrer and Gerd Binnig

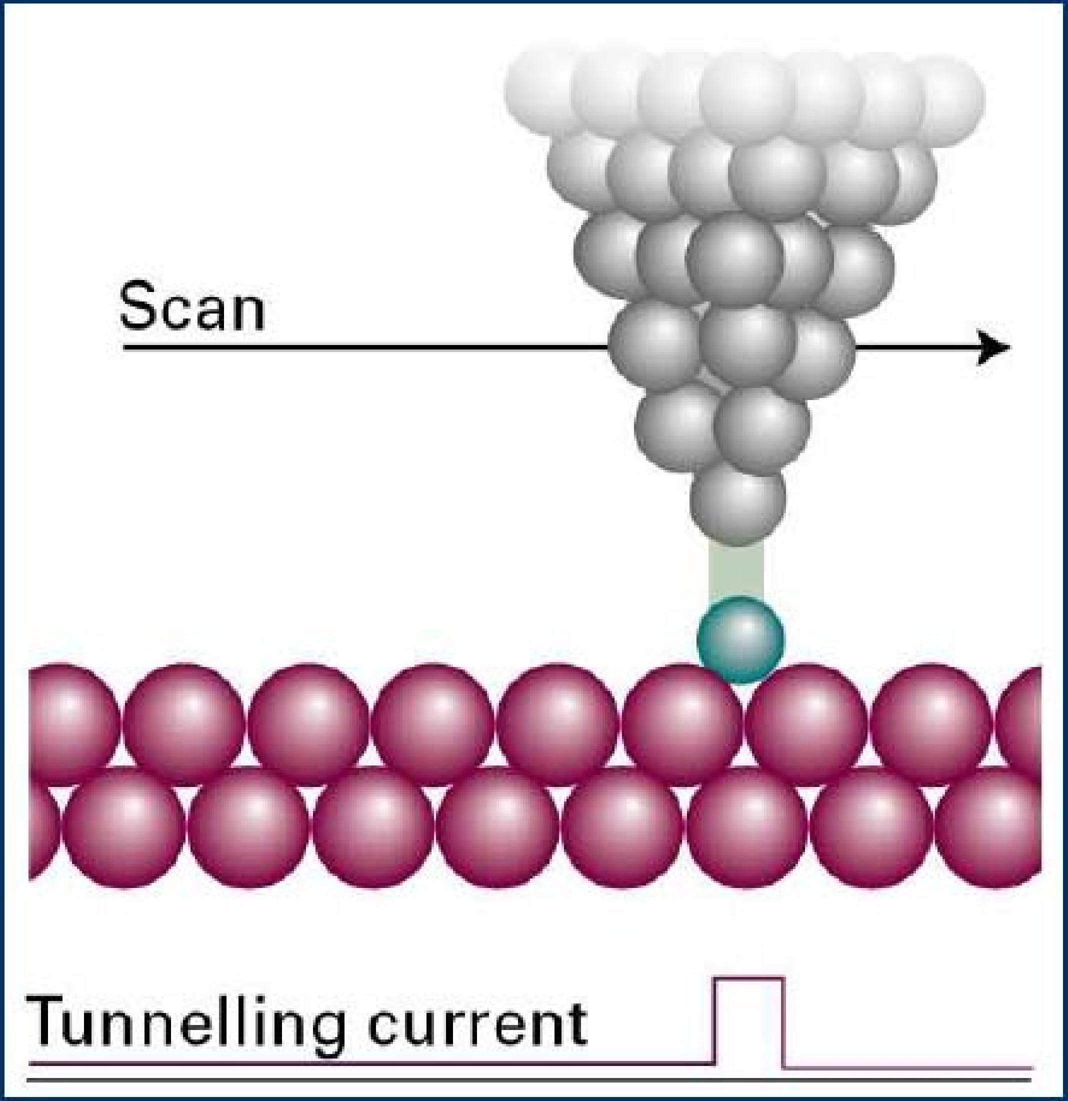
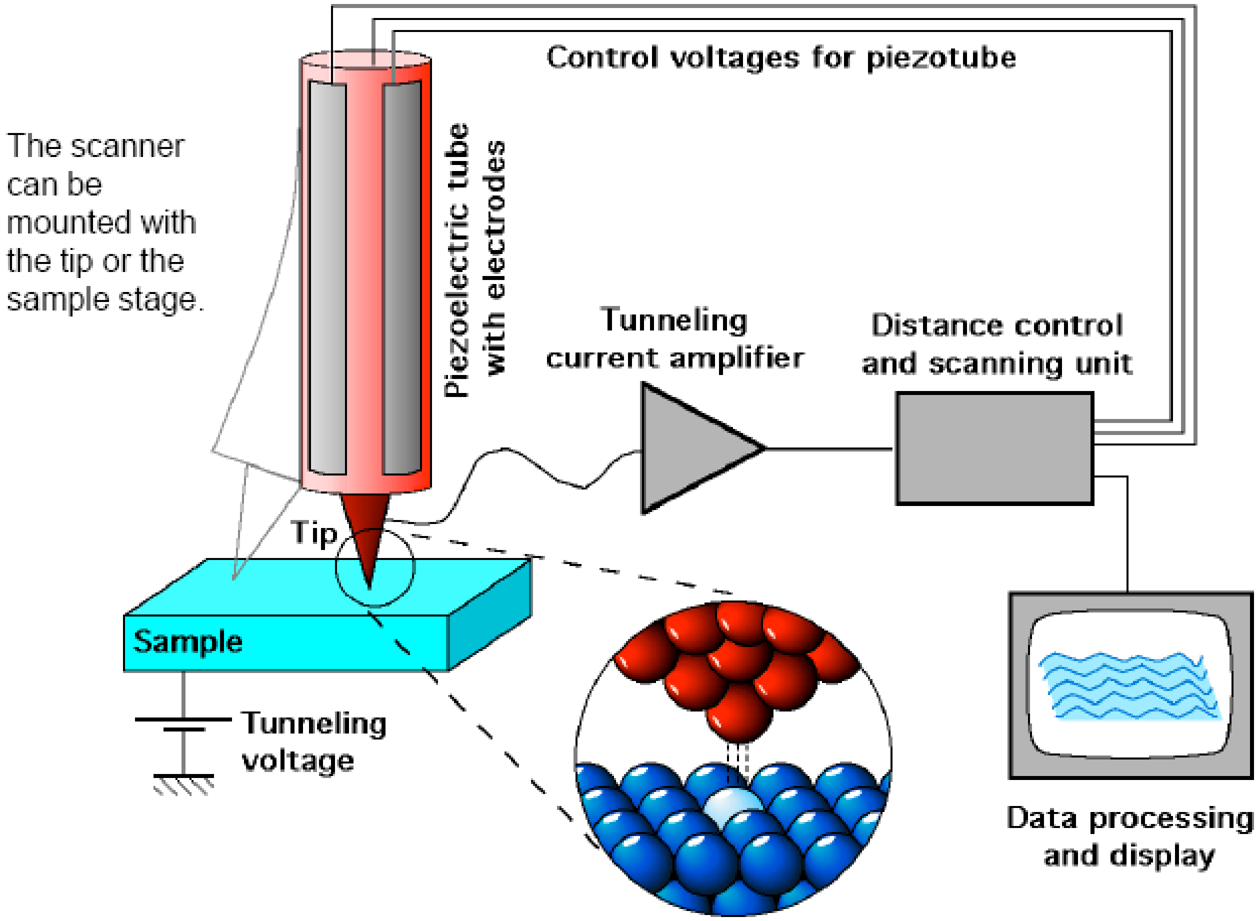


Fig. 8.15 A scanning tunnelling microscope makes use of the current of electrons that tunnel between the surface and the tip. That current is very sensitive to the distance of the tip above the surface.

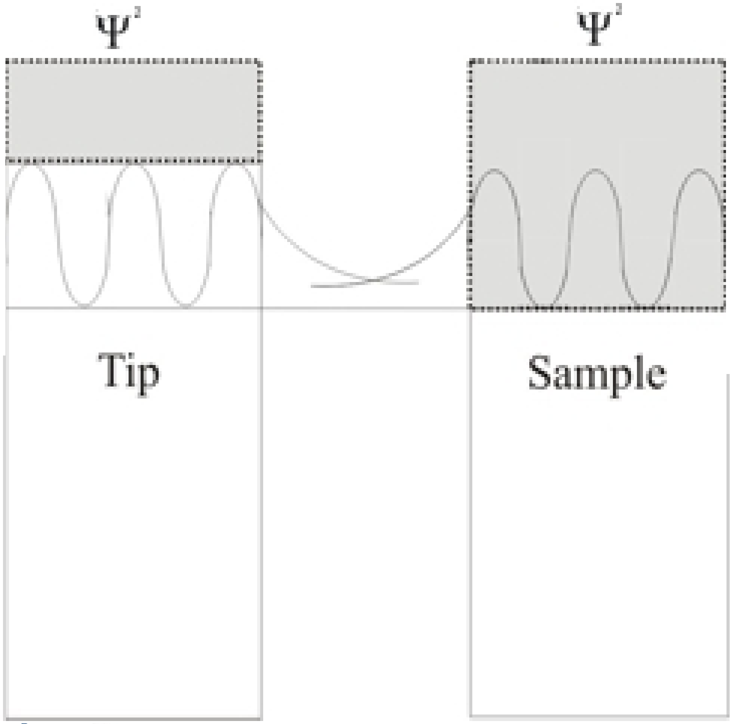
Basic components of STM:

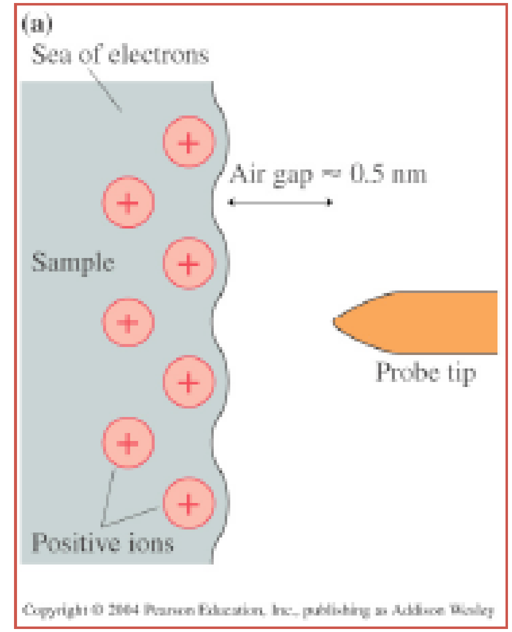


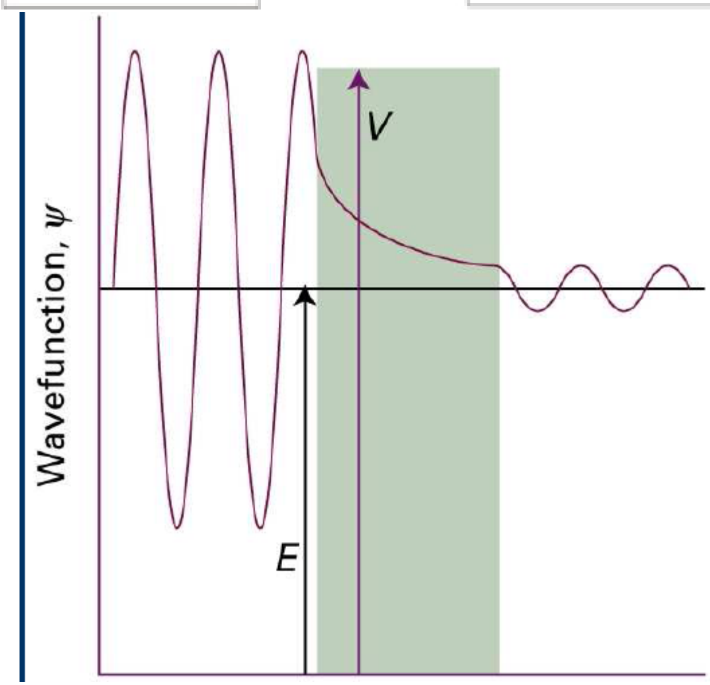
Five basic components:

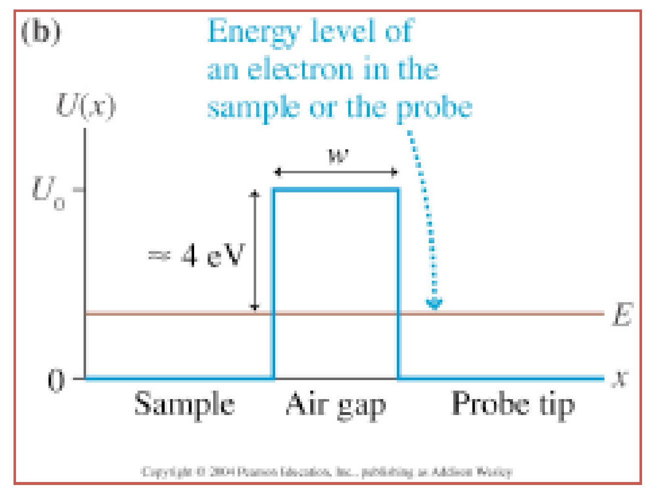
1. Metal tip,
2. Piezoelectric scanner,
3. Current amplifier (nA),
4. Bipotentiostat (bias),
5. Feedback loop (current).

A small air gap between the probe and the sample acts as the energy barrier!

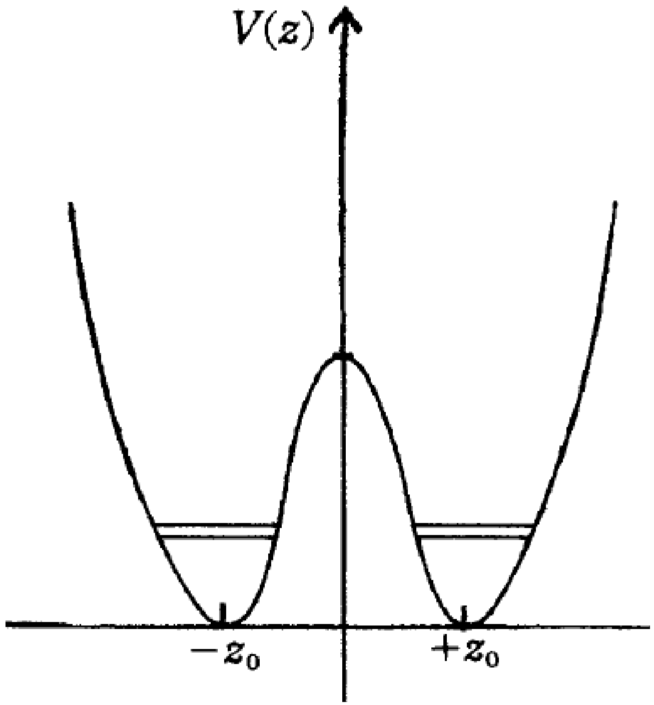

Ψ²
Ψ²
Tip
Sample

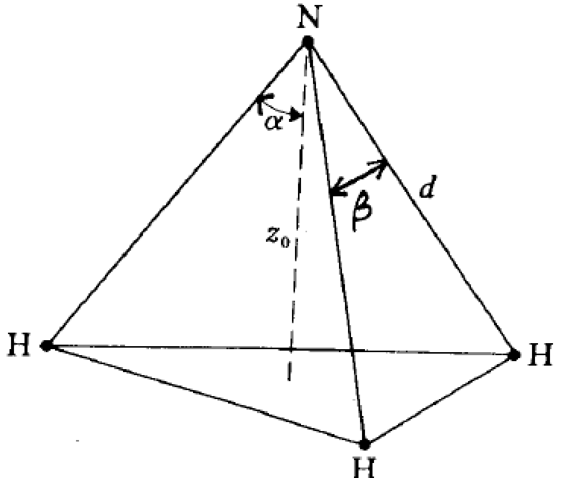

(a)
Sea of electrons
Air gap ≈ 0.5 nm
Sample
Probe tip
Positive ions

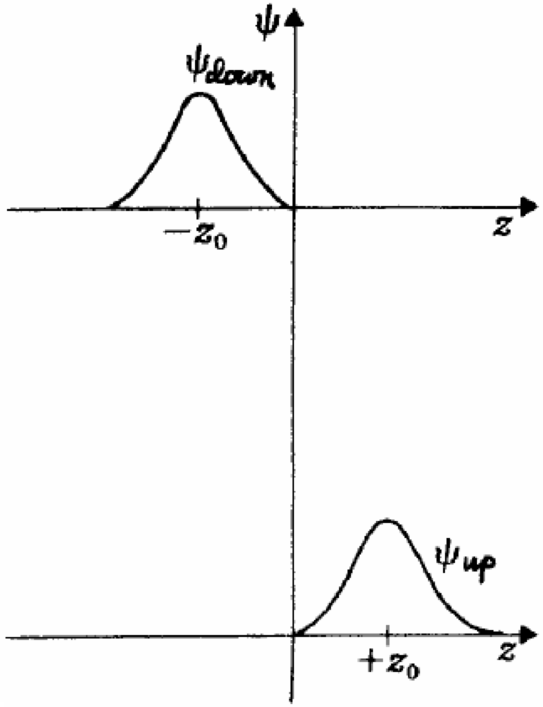

Wavefunction, ψ
V
E


(b)
Energy level of
an electron in the
sample or the probe
U(x)
w
U0
≈ 4 eV
E
0
x
Sample
Air gap
Probe tip

Ammonia Inversion

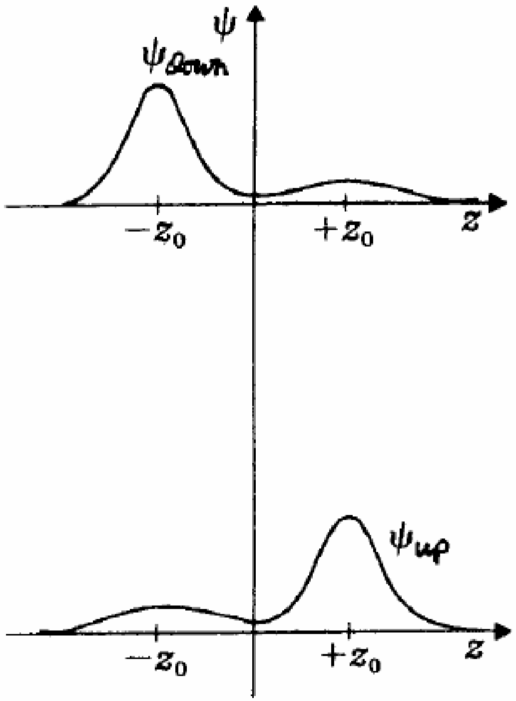






(a)

Infinite Potential



(b)

Finite Potential

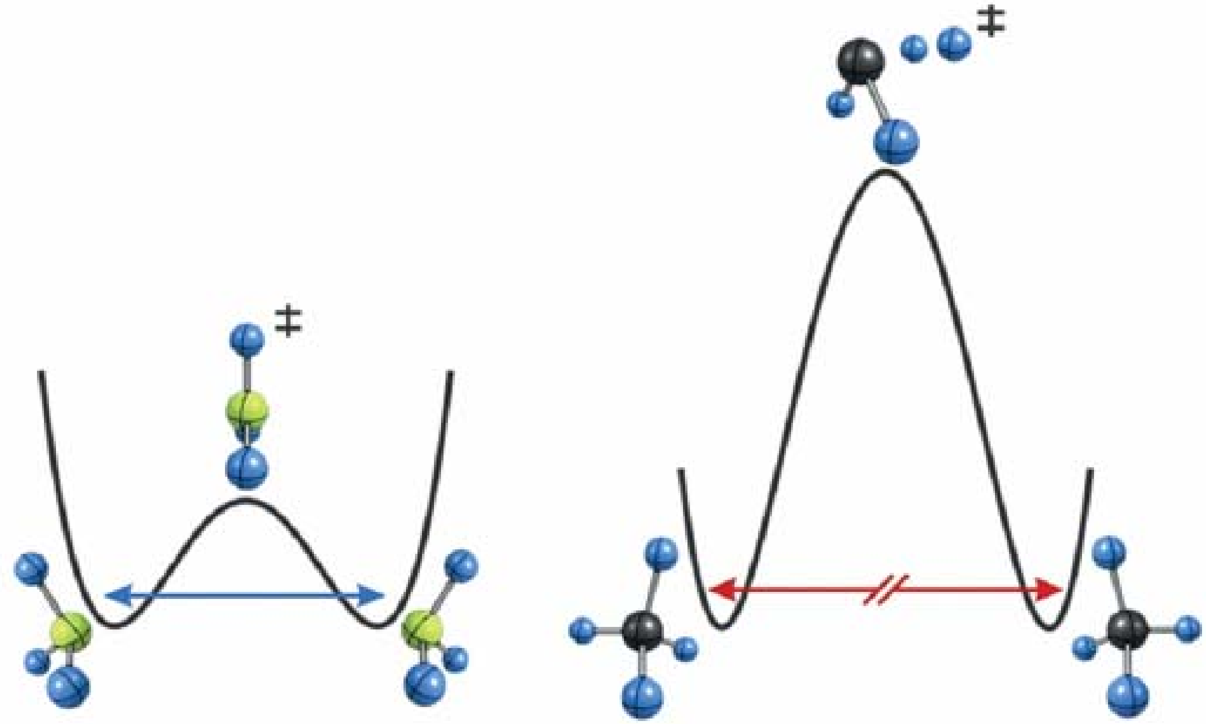
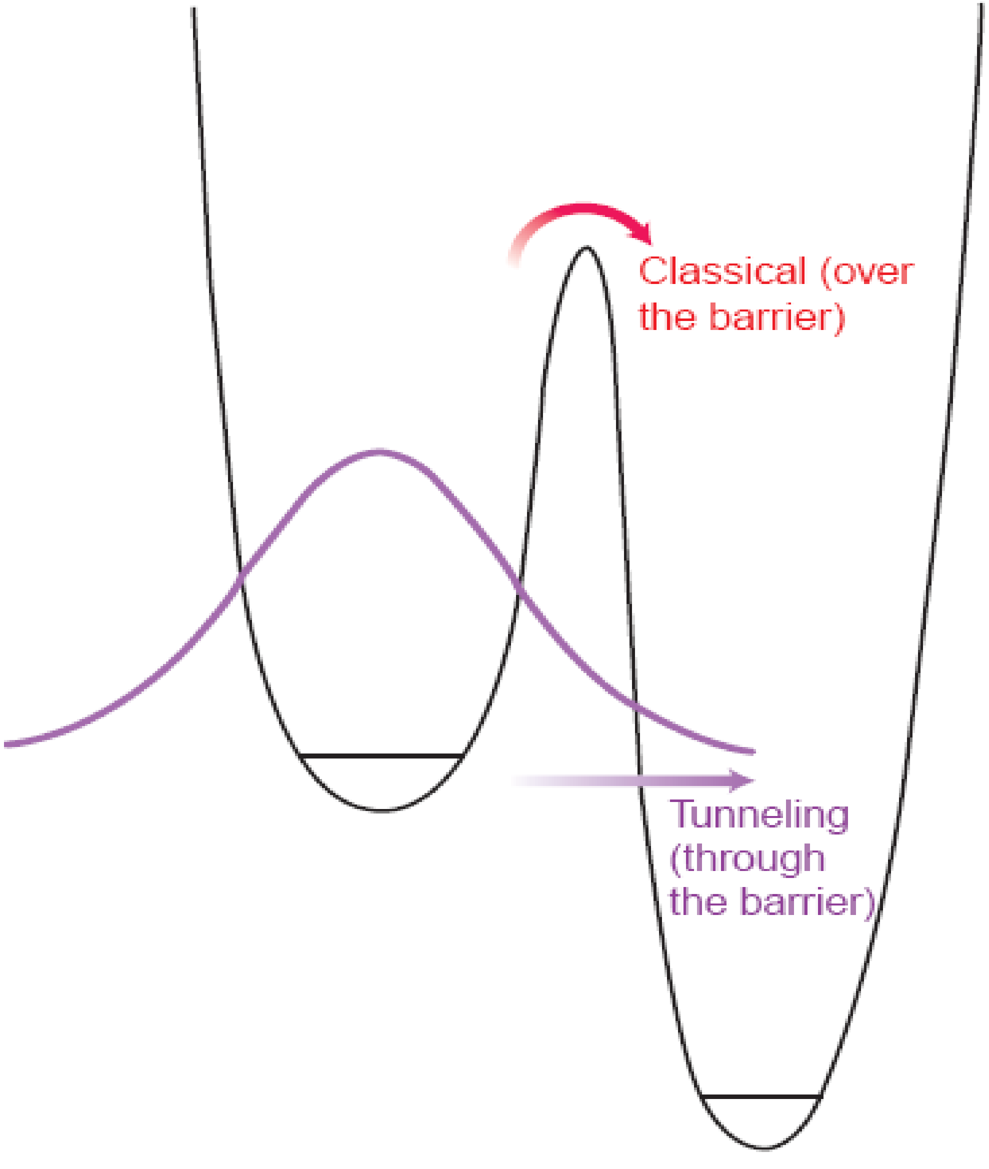


Fig. 2 The inversion has a strong tunnelling contribution in NH_3, but not so for methane.


Classical (over the barrier)
Tunneling (through the barrier)